An Appraisal of Methods for Computation of the Dispersion Characteristics of Open Microstrip

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Abstract—**A number of methods in the literature for computing the dispersion characteristics of open microstrip are compared in the case when substrate thickness is comparable to strip width. Significant discrepancies between the various results are found, and several suggestions are made to explain them.**

I. Introduction

A METHOD has recently been derived by the present authors [1] for the computation of the dispersion relation of open microstrip with electrically narrow strip width. In an effort to determine how wide a strip can be accurately described by this theory, a numerically "exact" solution was sought in the literature for purposes of comparison. Instead of finding such a standard, the authors discovered a large number of procedures [2]–[10], numerous computed results, but with comparisons between them rather sparse. In an attempt to evaluate these methods, their results have been compared where identical or similar microstrip parameters were used, and the results presented in graphical form. Mutual discrepancies of up to 25 percent in the effective permittivity (relative to the difference between this effective permittivity and its value at zero frequency) are noted. An attempt is made to explain this comparison on the basis of the functions used to represent the transverse current distribution in the various methods.

II. Comparison of Existing Methods

The integral equations describing the fundamental mode can be cast in a variety of forms; we follow the approach taken in [1]. From there we have that, for an assumed propagation factor of exp $(i\omega t - ik_0\alpha x)$, with α an as yet unknown normalized propagation constant, and x the distance along the strip axis (Fig. 1):

$$\int_{-l}^{l} G_e(y-y')\rho_1(y')dy' = \cosh\sqrt{\alpha^2-1}\; k_0 y \qquad (1)$$

where $\rho_1(y)$ is the charge distribution on the strip $(-l \leq y \leq l)$,

Manuscript received May 1, 1978; revised October 9, 1978. This work was supported by the Office of Naval Research under Contract N0014-76-C-0318; H. Mullaney contract monitor.

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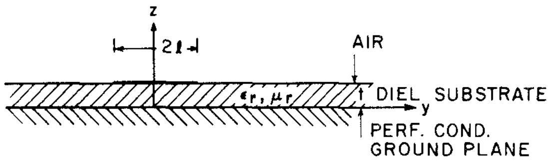

Fig. 1. Geometry of open microstrip.

$$G_e(y) = 2\int_0^\infty \frac{(u_n \tanh u_n T)\cos k_0\lambda y}{\epsilon_r u_0 + u_n \tanh u_n T}\,\frac{d\lambda}{u_0}. \qquad (2)$$

$T = k_0 t$ is the substrate thickness normalized to the free-space wavenumber, and

$$u_n = (\lambda^2+\alpha^2-\mu_r\epsilon_r)^{1/2} \quad u_0 = (\lambda^2+\alpha^2-1)^{1/2} \quad \mathrm{Re}\,(u_0) \geq 0 \qquad (3)$$

where ϵ_r and μ_r are, respectively, the relative permittivity and permeability of the substrate. Once the solution of (1) is known (as a function of α), the longitudinal current density $J_x(y)$ is then found from

$$\int_{-l}^{l} G_m(y-y')J_x(y')dy' = \frac{i\alpha}{k_0}\left[\cosh\sqrt{\alpha^2-1}\; k_0 y + \int_{-l}^{l} M(y-y')\rho_1(y')dy'\right] \qquad (4)$$

where

$$G_m(y) = 2\mu_r \int_0^\infty \frac{\cos k_0\lambda y\, d\lambda}{\mu_r u_0 + u_n \coth u_n T} \qquad (5)$$

$$M(y) = 2(\mu_r\epsilon_r - 1) \cdot \int_0^\infty \frac{\cos k_0\lambda y\, d\lambda}{(\epsilon_r u_0 + u_n \tanh u_n T)(\mu_r u_0 + u_n \coth u_n T)u_0}. \qquad (6)$$

The solutions $\rho_1(y)$ and $J_x(y)$ thus obtained (which are both dependent on α) are then inserted into

$$\int_{-l}^{l} [ik_0\alpha J_x(y) + \rho_1(y)]dy = 0 \qquad (7)$$

(which follows from the requirement that the transverse current density vanish at the edges of the strip) to obtain the characteristic equation for obtaining α.

In [4], [6]–[8], and [10], this system of equations (or one equivalent to it) is solved by expanding the unknowns $\rho_1(y)$ and $J_x(y)$ (or their counterparts) in terms of a set of basis functions, thereby converting the integral equations into an infinite set of algebraic equations for the coefficients in these expansions.[1] This system is then solved by truncation to a convenient number of unknowns. These methods, assuming no difficulties arise with relative convergence [11], will in principle converge to the correct value of α as a larger number of unknown coefficients is retained in the truncation.

However, [1]–[3], [5], and [9], on the other hand, all assume or derive some form of current and charge distributions to insert into (1)–(7) in order to obtain an approximate but closed form characteristic equation to solve for α. In the case of [5], [9], a variational formulation was used in an attempt to minimize the error due to this approximation. All these methods can be thought of as a very special case of the truncation method,[2] wherein only a single basis function and unknown coefficient is used for each of ρ_1 and J_x.

Since for sufficiently narrow strips or low frequencies the transverse current J_y will be small, it is reasonable to use the same basis function $I(y)$ for ρ_1 as for J_x, especially as both will have the same singularity at the edges of the strip. If we then take

$$\rho_1(y)=a_0(\alpha)I(y) \quad J_x(y)=b_0(\alpha)I(y) \tag{8}$$

and substitute into (1), (4), and (7), and eliminate the cosh term between (1) and (4), we obtain

$$2\mu_r\int_0^\infty \frac{\tilde{I}(\lambda)\cos k_0\lambda_y}{\mu_r u_0+u_n \coth u_n T}d\lambda = \alpha^2\left\{2\int_0^\infty \frac{\tilde{I}(\lambda)u_n \tanh u_n T \cos k_0\lambda y}{\epsilon_r u_0+u_n \tanh u_n T}\frac{d\lambda}{u_0}+2(\mu_r\epsilon_r-1)\cdot\int_0^\infty \frac{\tilde{I}(\lambda)\cos k_0\lambda y d\lambda}{(\epsilon_r u_0+u_n\tanh u_n T)(\mu_r u_0+u_n\coth u_n T)u_0}\right\} \tag{9}$$

where

$$\tilde{I}(\lambda)=\int_{-l}^{l} I(y)\cos k_0\lambda y dy=2\int_0^l I(y)\cos k_0\lambda y dy \tag{10}$$

is the Fourier transform of $I(y)$, assumed here to be an even function.

If we set $y=0$ in (9), and use the known charge distribution $(l^2-y^2)^{-1/2}$ on a strip in free space for $I(y)$ (so that $\tilde{I}(\lambda)=J_0(k_0\lambda l)$) we obtain the formula of Denlinger [2] and of Schmitt and Sarges [3]. Denlinger [2] also used the charge distribution $1+|y/l|^3$ as an approximation to $(l^2-y^2)^{-1/2}$ to simplify computation of $\tilde{I}(\lambda)$. If we set $y=2l/3$, we get a modified form of this result suggested by Kowalski and Pregla [5].

If the substitutions (8) truly represented the zeroth-order step of an expansion in a (presumably orthogonal) set of basis functions, instead of the procedure leading to (9), we would, after substituting (8) into (1), (4), and (7), next multiply (1) and (4) by $I(y)$ and integrate from $-l$ to l. The effect of this would be to replace $\tilde{I}(\lambda)\cos k_0\lambda y$ by $[\tilde{I}(\lambda)]^2$. This is then the result of Pregla and Kowalski [9], who obtained this result through the use of a variational functional of the current distribution. In particular, if $\tilde{I}(\lambda)=J_0(k_0\lambda l)$, we get the formula of [9], and also the formula of [4] if the Gaussian–Chebyshev quadrature is limited to one term. The zeroth-order expression of Itoh and Mittra [6] uses a constant basis function, whence it is our $[\tilde{I}(\lambda)]^2$ functional using $\tilde{I}(\lambda)=\sin(k_0\lambda l)/(k_0\lambda l)$. As for the remaining methods, the basis functions used in [7], [8] are not precisely specified; however, it appears from graphs in these papers that they might be $T_m(y/l)(l^2-y^2)^{-1/2}$, where T_m are the Chebyshev polynomials of the first kind, making this technique closely related to that of [4]. Farrar and Adams [10] use pulse functions as commonly found in moment-method applications; their zeroth-order approximation would be the same as that of Itoh and Mittra [6].

Finally, it should be mentioned that the approximation of the present authors for narrow strips [1] can be obtained from (9) by adding and subtracting the limiting forms of the integrands (except for $\tilde{I}(\lambda)\cos(k_0\lambda y)$) for large λ, e.g.,

$$\frac{1}{\mu_r u_0+u_n\coth u_n T}=\frac{1}{\lambda(\mu_r+\coth\lambda T)}+\left[\frac{1}{\mu_r u_0+u_n\coth u_n T}-\frac{1}{\lambda(\mu_r+\coth\lambda T)}\right]. \tag{11}$$

The first term, together with the current distribution $\tilde{I}(\lambda)=J_0(k_0\lambda l)$, can be integrated in closed form, and gives a term independent of frequency which, when approximated for $l^2\ll 4t^2$, reduces to the static inductance (or capacitance) term of [1]. The second term from (11) falls off rapidly with λ, and if we put $\tilde{I}(\lambda)\cos(k_0\lambda y)=1$ in these terms, then (9) reduces to the result of [1], which is considerably simpler than (9) and can be rapidly solved by iterative or other techniques since the dominant α-dependence is the factor α^2 in the right-hand side. It should be emphasized, however, that this result is strictly valid only in the narrow-strip limit.

A comparison of the numerical results for $\epsilon_{r_{\mathrm{eff}}}=\alpha^2$ of [1]–[10] is made in Figs. 2 and 3. Fig. 2 displays results[3] of [1]–[3], [6]–[10], all for identical configurations, except for [3] which used a slightly smaller ϵ_r. Fig. 3 compares a different case from [3]–[5], again all identical except for [5], which had a somewhat larger ratio of $2l/t$. Even though all results except those of [1] and [9] had to be

[1]The Gaussian–Chebyshev quadrature used in [4] to evaluate the integrals is essentially equivalent to expanding ρ_1 and J_x in terms of the Chebyshev polynomials.

[2]The method of [5] does not strictly fit this pattern, but the present authors have verified that the later formula [9] of the same two authors, which *does* fit the pattern, gives identical results for reasonably wide strips.

[3]See footnote 2. The formula of [9] was programmed by the present authors and results presented here.

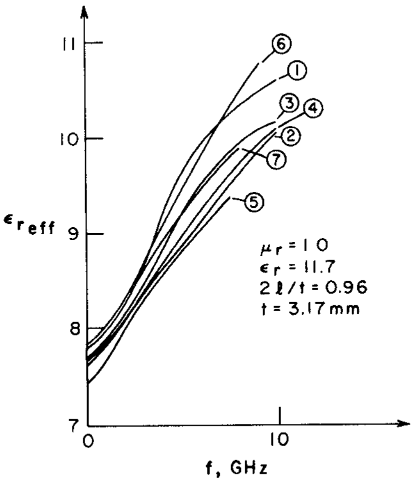

Fig. 2. Comparison of effective dielectric constant $\epsilon_{r_{\text{eff}}}=\alpha^2$ as computed by various authors. Parameters are as shown in figure, unless otherwise noted: ① Farrar and Adams [10]. ② Itoh and Mittra [6]. ③ Van de Capelle and Luypaert [7], [8]. ④ Denlinger [2]. ⑤ Schmitt and Sarges [3] ($\epsilon_r=11.2$). ⑥ Chang and Kuester [1]. ⑦ Pregla and Kowalski [9].

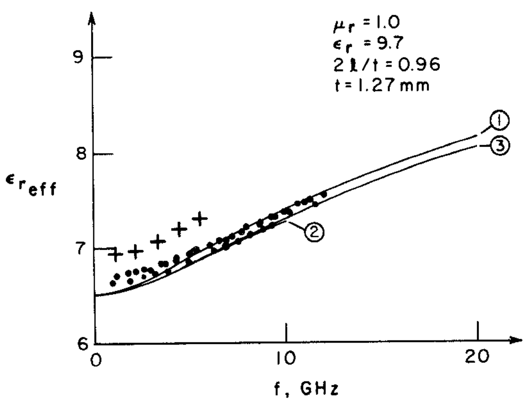

Fig. 3. Comparison of effective dielectric constant $\epsilon_{r_{\text{eff}}}=\alpha^2$ as computed by various authors. Parameters are as shown in figure, unless otherwise noted: ① Schmitt and Sarges [3]. ② Fujiki *et al.* [4]. ③ Kowalski and Pregla [5] ($2l/t=1.0$).

Experimental results: • Hartwig *et al.* [13]. + Deutsch and Jung [14] ($\epsilon_r=9.8$, $2l/t=1.0$).

read from graphs with an attendant and unavoidable error, there is such a spread, especially in Fig. 2, as could not be attributed to this error alone. Also included in Fig. 3 are results of measurements by Hartwig *et al.* [13] and by Deutsch and Jung [14]. The latter seem to have a significant constant discrepancy with all the theories, while the former are in rather close agreement, although none of the individual theories seems to be especially favored.

III. Discussion

Good agreement among the results of [7]–[9] is seen in Fig. 2, and between [4] and [5] in Fig. 3. The other solutions have deviations from these and each other; in Fig. 2 these are as large as 25 percent of the total dispersion of $\epsilon_{r_{\text{eff}}}$ (that is, $\epsilon_{r_{\text{eff}}}-\epsilon_{r_{\text{eff}}}|_{f=0}$) at $f=8$ GHz. While it is beyond the scope of this short note to program the methods of [1]–[10] and perform exhaustive comparisons among them to determine a "best" technique for the problem, several possible reasons for the discrepancies can be offered.

First, it can be noted that the basis functions of [6] and [10] do not have the appropriate edge singularity for $\rho_1(y)$ or $J_x(y)$ at $y=\pm l$. As a consequence, a very large number of terms in these expansions might be necessary in order to minimize the error which results. Second, the formulas of [2] and [3], while the edge singularity has been included, have chosen arbitrarily to satisfy (9) at $y=0$. We have verified that the (again, arbitrary) choice of $y=2l/3$ suggested in [5] can result in a change in $\epsilon_{r_{\text{eff}}}$ of 10–15 percent at $f=8$ GHz for the typical cases considered here, i.e., when the strip is not narrow compared with the substrate. This potential source of error is avoided in the other methods by the multiplication of (9) by $I(y)$ and integration from $-l$ to l. As pointed out in [9], this also has the advantage that a single basis function approximation is additionally a variational expression, and thus potentially more accurate. Finally, the accuracy with which the integrals in (9) are evaluated numerically is a possible source of error. This seems a possible explanation for the discrepancy between the results of [2] and [3] given in Fig. 2, which seems larger than can be attributed solely to the different values of ϵ_r.

The paper of Jansen [12] appeared after this comparison had been completed. Jansen notes the necessity for the basis functions to satisfy the edge condition, and also notes that basis functions, such as those used in [6] and [10] which are not continuously differentiable, can also lead to extraneous, nonphysical solutions. The basis functions used in [12] are similar to those used in [4] (although the author did not refer to this paper or make explicit comparisons with any other results). The results presented in [12] for narrow strips do not extend to substrates which are very large electrically, but for $t=0.64$ mm, $2l/t=0.9375$, $\epsilon_r=9.9$, and $f=16$ GHz, a result of $\epsilon_{r_{\text{eff}}}=7.25$ is obtained. This result would remain the same if f and t were scaled to 8.06 GHz and $t=1.27$ mm, respectively, $2l/t$ remaining constant. The closest comparable result in [4] is for $t=1.27$ mm, $2l/t=0.96$, and $\epsilon_r=9.7$, which is $\epsilon_{r_{\text{eff}}}=7.115$ (this is the result plotted in Fig. 3). If this result can be scaled up for $\epsilon_r=9.9$ by simply multiplying by the ratio (9.9/9.7), the agreement can be seen to be excellent.

In summary then, we would speculate that the techniques of [4], [5], [7]–[9], and [12] are the most accurate of those considered here, for the case when the strip width is comparable to the substrate thickness. The methods of [6] and [10] can in principle also be made accurate in this range, but because the basis functions do not as precisely represent the actual behavior of the current and charge, a larger number of approximating functions is probably needed than is reported therein. The theories of [1]–[3], on the other hand, are fundamentally narrow-strip approximations, and can only give qualitatively correct results in the dispersive regime. It would appear that a more detailed examination of these theories is needed before any of their results can be used as a standard for comparison when $2l/t \simeq 1$.

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Accurate Solution of Microstrip and Coplanar Structures for Dispersion and for Dielectric and Conductor Losses

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***Abstract*—For the analysis of coplanar- and microstrip-type structures, a higher order solution of the spectral-domain approach is introduced. Legendre polynomials are used as the basis functions for fields having singularities near the edges, leading to fast convergence to the exact solution. A perturbation technique is combined with the spectral-domain method to evaluate conductor and dielectric losses in microstrip, inverted microstrip, and coupled microstrip in the metallic enclosure. Computations of characteristic impedance and losses incurred in several structures are also presented. Central processing unit (CPU) time on an IBM 360/65 for the zeroth-order approximation ranges from 1 to about 5 s for the whole computation, and increases if higher order of solution is requested for better accuracy. The calculation of attenuation due to conductor losses is found to be particularly sensitive to order of approximation, so that the generally used "zeroth-order" solution is inadequate. A user-oriented program package has been written, including options on order of mode, order of solution (i.e., of approximation), impedance, attenuation, and number of substrates. Although written for single or coupled microstrip, the program can be adapted for arbitrary arrangements of thin coplanar conductors. The program is described separately.**

Manuscript received August 14, 1978; revised March 28, 1979.

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I. Introduction

THE WIDESPREAD use of MIC's in recent years has caused rapid progress in the theory and technology of it. The very first transmission line used in MIC was, indeed, microstrip laid on the dielectric substrate, and then other transmission lines such as slot line, suspended microstrip, and so on, were introduced and improved.

Initially, the analysis for this class of transmission line was invariably a quasi-TEM approximation, except for slot line where Cohn [1] introduced a frequency dependent solution because of its different nature. Although a quasi-TEM solution at low frequency can yield satisfactory results, at high frequency its weakness becomes apparent. To feature the frequency dependence of these lines, one must consider a hybrid mode analysis which in turn is more tedious, and in some cases requires enormous computing time. This dispersion analysis was studied by various workers and by various methods. For instance,

0018-9480/79/0700-0694$00.75